Day 5 - Review Extra Problems AK

① $\int (6x^2 - 9x + 3)\,dx$

$y = 2x^3 - \frac{9}{2}x^2 + 3x + c$

② $\int 3x\sqrt{2x^2+3}\,dx$ $u = 2x^2 + 3$

$\int 3x\, u^{1/2}\, \dfrac{du}{4x}$ $du = 4x\,dx$

$\int \frac{3}{4} u^{\frac{1}{2}}\,du = \frac{3}{4}\cdot\frac{2}{3} u^{3/2} + c \Rightarrow y = \frac{1}{2}(2x^2+3)^{3/2} + c$

③ $\int -4\sec(x)\tan(x)\,dx$

$y = -4\sec(x) + c$

④ $\int \dfrac{2x - 3x^{1/2} + 5}{x^{1/3}}\,dx$

$\frac{1}{2} - \frac{1}{3} \to \frac{3}{6} - \frac{2}{6}$

$\int\left(\dfrac{2x}{x^{1/3}} - \dfrac{3x^{\frac{1}{2}}}{x^{1/3}} + \dfrac{5}{x^{1/3}}\right)dx = \int\left(2x^{2/3} - 3x^{1/6} + 5x^{-1/3}\right)dx$

$= \frac{6}{5}x^{5/3} - \frac{18}{7}x^{7/6} + \frac{15}{2}x^{2/3} + c$

⑤ $\int 4x^2 \csc^2(2x^3)\,dx$ $u = 2x^3$

$\int 4x^2 \csc^2(u)\,\dfrac{du}{6x^2}$ $du = 6x^2\,dx$

$\int \frac{2}{3}\csc^2(u)\,du$

$y = -\frac{2}{3}\cot(u) + c$

$y = -\frac{2}{3}\cot(2x^3) + c$

⑥ $\int \sec^3(x)\tan(x)\,dx$ $u = \sec(x)$

$\int \sec^2(x)\cdot\sec(x)\tan(x)\,dx$ $du = \sec(x)\tan(x)\,dx$

$\int u^2 \sec(x)\tan(x)\,\dfrac{du}{\sec(x)\tan(x)}$

$\int u^2\,du$

$y = \frac{1}{3}u^3 + c$

$y = \frac{1}{3}\sec^3(x) + c$

⑦ $\int \dfrac{12x - 10}{\sqrt{3x^2 - 5x}}\,dx$ $u = 3x^2 - 5x$
$du = (6x - 5)\,dx$

$\int \dfrac{2(6x-5)}{u^{1/2}}\,\dfrac{du}{(6x-5)}$

$\int 2u^{-1/2}\,du = 4u^{1/2} + c$

$y = 4(3x^2 - 5x)^{1/2} + c$

⑧ $\int \dfrac{1}{\sqrt{x}}\cos(\sqrt{x})\,dx$ $u = \sqrt{x}$
$u = x^{1/2}$
$du = \frac{1}{2}x^{-1/2}\,dx$

$\int x^{-1/2}\cdot\cos(u)\,\dfrac{du}{\frac{1}{2}x^{-1/2}}$

$\int 2\cos(u)\,du$

$y = 2\sin(u) + c$

$y = 2\sin(\sqrt{x}) + c$

⑨ $\int dx$

$y = x + c$

⑩ $f'(x) = 6x - 5$ $f(1) = 4$

$\int y' = \int (6x - 5)\,dx$

$y = 3x^2 - 5x + c$

$4 = 3(1)^2 - 5(1) + c$

$4 = -2 + c$

$6 = c$

$\boxed{y = 3x^2 - 5x + 6}$

⑪ $f'(x) = 3x(x^2 - 4)^3$ $f(2) = 7$

$\int y' = \int 3x(x^2 - 4)^3\,dx$ $u = x^2 - 4$

$\int y' = \int 3x\cdot u^3\,\dfrac{du}{2x}$ $du = 2x\,dx$

$\int y' = \int \frac{3}{2}u^3\,du$

$y = \frac{3}{8}u^4 + c$

$y = \frac{3}{8}(x^2 - 4)^4 + c$

$7 = \frac{3}{8}\left((2)^2 - 4\right)^4 + c$

$7 = c$

$\boxed{y = \frac{3}{8}(x^2 - 4)^4 + 7}$

$|y = 8x^2 - 5x + 6|$

$y = \frac{3}{8}(x^2 - 4)^4 + c$

$7 = c$

$\boxed{y = \frac{3}{8}(x^2 - 4)^4 + 7}$